As 2011 draws to a close, I find myself pondering a simple investment idea - portfolio rebalance. Before embarking on any New Year investment resolutions or acting on so-called top 10 investment ideas of 2012, the year end is a good time for investors to consider rebalancing portfolios to their prescribed weights. Why is portfolio rebalancing so important? As far as I know, it is the simplest and clearest technique that, with few exceptions, adds incremental value to fixed-weighted multi-asset class portfolios. This incremental value is often referred to as diversification return.

In one of my recent papers on Risk Parity, I used the following example to illustrate the power of diversification. “Suppose we have at our disposal two investments. Investment A doubles in the first year and then promptly drops by half in value in the second year. In contrast, investment B moves in the opposite way: it goes down by 50% in year one and then recovers by 100% in year two. Both investments have gone nowhere individually, after two tumultuous years. However, a 50/50 portfolio with 50% of capital in each would yield a return of 25% in both year one and year two (rebalancing!) without annual return volatility.”

Soon after the paper’s publication, one reader emailed me a question regarding the interpretation of this example. The question was whether the positive return was due to diversification or due to rebalancing. This is a good question. Without rebalancing, any portfolio, regardless of its initial weights, would not have any gain. Similarly, a concentrated portfolio (e.g. 100% in either investment A or B) would be devoid of any rebalancing opportunity and also yields zero return. Among all possible portfolios of the two assets, the 50/50 portfolio is the most diversified in terms of generating the highest diversification return through rebalance. So my answer to his question was that diversification and rebalance must go hand in hand, thus making it hard to separate their contributions.

Portfolio rebalancing is a well-known technique and an old research topic. For instance, numerous articles have been written regarding different rebalancing techniques: calendar rebalance, threshold rebalance, tactical rebalance, etc. So why should we spend more time on the topic? The reason is that all the previous research focuses almost exclusively on long-only portfolios without leverage. What happens when we have portfolios such as Risk Parity portfolios that are leveraged, or portfolios with both long and short positions? Does portfolio rebalance still make sense? Does it still generate incremental value? These are important questions as investors continue to embrace the concept of Risk Parity and appreciate the benefits of appropriately used leverage. In this research note I shall address these questions and share some topical insights. More details can be found in a forthcoming paper.

Rebalancing Long-Only Unlevered Portfolios

Before considering leveraged portfolios, let’s review some results concerning fixed-weighted, long-only, unlevered portfolios. For a calendar-based rebalancing scheme, the exercise is simple: periodically a portfolio is rebalanced back to its original weights. However, to understand it further and extend our
understanding to levered portfolios, we note that the rebalance is carried out as a contrarian, or mean-reverting, strategy.

In our simple example, the 50/50 portfolio drifts to an 80/20 portfolio (100/125 = 80%, 25/125 = 20%) after year one. In order to rebalance the portfolio to the original 50/50 weight, we sell 30% of asset A (the winner) and buy 30% of asset B (the loser). This mean-reverting strategy is also true for portfolios with more than two assets. The weights of the assets that have positive excess returns versus the portfolio will drift higher, while the weights of assets that have negative excess returns versus the portfolio will drift lower. As a result, rebalancing necessarily requires selling the former group (the winners) and buying the latter group (the losers).

Why would such a mean-reverting strategy generate positive value added for a long-only portfolio? This is because such a strategy takes advantage of randomness or volatility of asset returns, which is always present regardless of how different the average returns of the individual assets might be. In the simplest case, where all assets have identical cumulative returns over time, the mean-reverting strategy would sell assets that happen to overshoot the long-term average and buy assets that happen to undershoot their long-term average. As a result, the mean-reverting strategy generates positive diversification return versus the weighted sum of the individual returns.

Diversification Return

It is time we formally define diversification return as being the difference between the geometric return of a multi-asset class portfolio and the weighted sum of the geometric returns of the underlying asset classes. It is crucial to see that the geometric average return, not arithmetic average return, is what matters. Mathematically we define the diversification return as

$$r_d = g_p - \sum_{i=1}^{N} w_i g_i,$$

where $g$ denotes the geometric return.

This diversification return is always non-negative for fixed-weighted, long-only, unlevered portfolios, when returns follow lognormal distributions. I provide a mathematical proof in the upcoming paper. It suffices to state here that the key to the proof is that the variance of a fixed-weighted portfolio is, in general, smaller than the weighted sum of the individual variances because of diversification. This variance difference is approximately twice the diversification return defined above. As a consequence, the geometric return of the portfolio is greater than the weighted sum of the geometric returns of the individual assets. Indeed, the whole is bigger than the sum of its parts! In the case in which all individual assets have zero cumulative return, as in my two-asset example, portfolio rebalancing can generate a positive return over time – in other words, the proverbial making of something out of nothing.

For clarity, we shall continue to use portfolios of two assets as illustrations throughout this research note. Assuming portfolio weights $(w_1, w_2)$, volatilities $(\sigma_1, \sigma_2)$, and correlation $\rho_{12}$, the diversification return is given by:

$$r_d = 0.5(w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 - \sigma_p^2), \quad (1)$$
\[ \sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2 \rho_{12} \sigma_1 \sigma_2. \] (2)

Equation (1) states diversification return is half of the difference between the weighted sum of the two individual variances and the portfolio variance. Equation (2) gives the portfolio variance in terms of the weights and the variances and the correlation. For long-only unlevered portfolios, \(0 \leq w_1, w_2 \leq 1, w_1 + w_2 = 1\), it is always the case that \(\sigma_p^2 \geq 0\). We use this result to analyze a 60/40 portfolio with stocks and bonds. Assuming volatility of 20% and 5% for stocks and bonds respectively, the diversification returns would be 0.63%, 0.51%, and 0.39% for three different correlations: \(\rho = -0.5, 0, 0.5\). The results are summarized in the table below, together with three portfolio volatilities.

Exhibit 1 Diversification returns and volatilities of 60/40 portfolios

<table>
<thead>
<tr>
<th>60/40</th>
<th>(\rho = -0.5)</th>
<th>(\rho = 0)</th>
<th>(\rho = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diversification return</td>
<td>0.63%</td>
<td>0.51%</td>
<td>0.39%</td>
</tr>
<tr>
<td>Portfolio volatility</td>
<td>11.1%</td>
<td>12.2%</td>
<td>13.1%</td>
</tr>
</tbody>
</table>

For illustrative purposes only. Source: PanAgora.

Exhibit 1 shows that lower correlation not only leads to lower portfolio volatility, but also a higher diversification return. There are important implications of this diversification return analysis to asset allocation decisions between stocks and bonds. It is known that the correlation between stocks and bonds varies greatly when the bonds are of different credit quality. The correlation is typically negative when the bonds are high-quality government bonds. The correlation is near zero if one invests in aggregate bond indices. The correlation is highly positive if one invests in corporate bonds and high-yield bonds. In the last case, not only is the benefit of risk reduction small, but the high correlation also tends to decrease the diversification return.

**Rebalancing Leveraged Portfolios**

In contrast, rebalancing leveraged portfolios involves momentum, or trend following, i.e., it is buying winners and selling losers. While this might appear surprising and perhaps counter-intuitive, a simple example suffices to illustrate the logic.

Consider again a two-asset 200/100 portfolio with 200% long in asset A and 100% short in asset B. Suppose asset A returns 50% and asset B returns 0%. At the end of period, asset A grows to 300% and asset B stays at -100% so the net value of the portfolio doubles. As a result, the portfolio weight shrinks to 150/50 (300/200 = 150% and 100/200 = 50%). To rebalance the portfolio to the original 200/100 target weights, we would buy 50% of asset A (the winner) and sell 50% of asset B (the loser). It is easy to prove mathematically that when a leveraged portfolio has positive returns, gross leverage declines and one would have to increase leverage to get back to the original weights.

The opposite is true when a leveraged portfolio suffers losses. Suppose asset A returns -25% and asset B returns 0%. Then asset A declines to 150% and asset B stays at -100% so the net value of the portfolio is halved. As a result, the portfolio weight is now 300/200 (150/50 = 300% and 100/50 = 200%). To rebalance the portfolio to the original 200/100 mix, we would sell 100% of asset A (the loser) and buy (or short-covering) 100% of asset B (the winner). When a leveraged portfolio has negative returns, gross leverage increases and one would have to cut leverage or stop-loss to get back to the target weights. In this context, the practice of stop-loss or deleveraging, which is sometimes hailed as a distinctive investment decision by
hedge fund managers, at times when fresh capitals is not readily available, can be viewed merely as a mechanical decision to rebalance long-short portfolios.

So does this trend-following rebalance strategy generate diversification return? If it does, will the diversification return be negative? The answer is it is negative in some cases and positive in other cases.

**Negative Diversification Returns: Cautionary Tales of Inverse and Leveraged ETFs**

The simplest example of leveraged portfolios that have negative diversification returns (an oxymoron perhaps) is short portfolios, or leveraged portfolios of a single risky asset. Nowadays, one doesn’t have to look very long in order to find real-world examples. Over the last few years, there has been a proliferation of inverse and leveraged ETFs. The former typically takes short positions of cap-weighted indices while the latter takes leveraged exposures of cap-weighted indices, with the aid of derivatives. The positions or weights are rebalanced on a daily basis, hence the daily returns of these ETFs mirror the prescribed multiple of the underlying indices. But over the long run, they are guaranteed to lag the index multiples.

It is known by now that the daily rebalancing of these inverse and leveraged ETFs amounts to buying high and selling low. However, it is not widely recognized that the fundamental cause of the return slippage is the fact that portfolio rebalancing of these ETFs to constant leverage ratios generates a diversification return that is surely negative. We shall prove this fact below.

For portfolios with a single risky asset and the risk-free asset (i.e. cash) - a necessary component for inverse or leveraged portfolios, the formula for the diversification return reduces to

$$r_d = 0.5(w_1 - w_1^2)\sigma_1^2, \quad (3)$$

where $w_1$ is the weight of the risky asset and $\sigma_1$ is the volatility of the excess return of the risky asset. For an inverse index ETF, $w_1 = -100\%$. For a 2X ETF, $w_1 = 200\%$. It is easy to see both weights, when substituted into the equation, results in a negative diversification return. However, for long-only portfolios, in which $0 < w_1 < 100\%$, the diversification is always positive.

So how large is the potential return slippage? We provide numerical examples by assuming the risk index has an annual volatility of 20% (the likes of the S&P 500 index) and a daily volatility of about 1.3% ($= 20\% / \sqrt{250}$), assuming daily returns are independent. Using Equation (3) above, we obtain the daily return slippage for the following five ETFs, -3X, -2X, -1X, 2X, and 3X, shown in the second row of Exhibit 2 below. The next row displays the annualized slippage. The last two rows show the annual returns of the five ETFs when the underlying index return is -5% or 5% per year respectively. The annual slippage of -3X ETF is a whopping -21.3% while that of 3X ETF is -11.3%. The return slippage creates
significant return hurdles for these leveraged and inverse (or both) ETFs. For example, when the index annual return is -5%, a naïve investor might expect the super bearish -3X ETF to yield +15%, but in reality, the likely outcome (gross of fees) is negative, which is -6.3% (= 15% − 21.3%). On the other hand, an investor in a 3X ETF might dream of a 15% annual return when the index returns 5% in a year. The likely outcome (gross of fees) is only 3.7% (= 15% − 11.3%), which is actually less than the return of the plain vanilla index itself!

In my view, while these inverse and leveraged ETFs based on a single risky asset might be an efficient tool for short-term investments, they are ill-suited as long-term investments due to the large negative diversification returns\textsuperscript{iv}.

**Diversification Return of Leverage Risk Parity Portfolios**

We now turn to the diversification return of Risk Parity multi-asset class portfolios. The key insight is that the rebalance of a long-only, fixed-weighted, leveraged portfolio with *multiple risky asset classes* combines two separate strategies on the two separate levels analyzed above. The first level is bottom up, in which the mean-reverting strategy is carried out across the individual asset classes. The second level is top down, in which the trend following strategy is employed on the total portfolio that is leveraged. As we demonstrated above, the bottom-up mean-reverting strategy generates positive diversification return, which is amplified by the leverage in this case, while the top-down trend following strategy gives rise to negative diversification return. The net result, which can be either positive or negative, depends on the asset allocation mix as well as the overall leverage of the portfolio.

We use stock/bond portfolios as an example to illustrate the point. First, an unleveraged Risk Parity portfolio with stocks at 20% volatility and bonds at 5% volatility is a 20/80 portfolio. Exhibit 3 shows the diversification returns and portfolio volatilities of the 20/80 portfolios with three different correlations. Notice the volatilities are quite low because only 20% is allocated to stocks. It is also noted that the level of diversification return is lower than that of the 60/40 portfolios. This is not overly surprising since the maximum diversification return is achieved with the 50/50 equal-weighted portfolio, and 60/40 is closer to the 50/50 portfolio than 20/80.

When the correlation is zero, the required leverage declines to 215%. In this scenario, the diversification return stays at 0.34% and the bottom-up and the top-down effects caused by leverage are roughly equal. However, the diversification return of 0.34% is still lower than that of the 60/40 portfolio at 0.51%.
When the correlation is 0.5, the required leverage is only 190%. In this case, the leverage lowers the diversification return from 0.26% to 0.08% since the high correlation between the two assets limits the diversification return from the bottom-up effect.

Comparing the three cases, one concludes that low correlation among the two assets is key to maintaining a high level of diversification return as the underlying portfolio is being leveraged. From this perspective, high quality government bonds again serves as a better diversifier than corporate or high yield bonds.

Another noticeable fact of Exhibit 4 is that in all three cases, the diversification returns of the leveraged portfolios remain positive despite leverage. Of course, this can’t be true if the leverage ratios are much higher especially in the case of positive correlation. Therefore it is important that the leverage ratio be kept at levels that are consistent with a positive diversification return.

**Conclusion**

This research note provides some analytical results regarding portfolio rebalancing and the associated diversification returns for different kinds of portfolios. For long-only, unlevered portfolios we show that rebalancing amounts to mean-reverting strategies and the diversification return is always positive for multi-asset class portfolios. Therefore, adherence to regular portfolio rebalancing is strongly recommended.

For short (or inverse) and leveraged portfolios, we provide the key insight that rebalancing on the top down level amounts to trend-following strategies that detract diversification returns, while rebalance among individual assets at the bottom-up level is still mean-reverting and adds to the diversification return.

We highlight the pitfalls of inverse and leveraged ETFs based on a single risky asset, for instance a cap-weighted equity index, which often carries a significant negative diversification return, or return slippage over time, against index return multiples.

Skeptics might argue that cap-weighted indices are not exactly a single risky asset. It is imperative to realize that even though cap-weighted indices might consist of hundreds, or even thousands, of securities, they are not fixed-weighted portfolios and hence offer no diversification return since cap-weighted indices never rebalance. In fact, this might be the Achilles’ heel of cap-weighted indices, apart from their risk concentration in countries, sectors, or stocks. Without regular rebalance, frequent booms and busts of individual stocks, sectors, and countries within the indices simply take investors on wild rides that go nowhere over time, not too dissimilar in spirit to portfolios of investments A and B in our simple example, when not rebalanced. Viewed in this light, equal-weighted indices might seem to be naïve, but regular rebalancing at least provides a guaranteed positive diversification return over time. This crucial difference goes a long way to explain why equal-weighted indices often beat cap-weighted indices in the long run.

Our results concerning Risk Parity portfolios show that the diversification return is in general positive for leveraged Risk Parity portfolios when the leverage ratio is not too high (under 300%) for two asset class portfolios with stocks and bonds. For portfolios with more asset classes, the leverage ratios can still be higher due to the fact that the bottom-up diversification effect will be higher with more assets.
Another important factor in determining portfolio diversification return is the correlation between different assets. The lower the correlation is, the higher the diversification return. Our stock/bond example strongly suggests that the best diversifying assets to stocks are high-quality government bonds due to their negative correlation to stocks. In contrast, the worst diversifying assets to stocks might be high yield corporate bonds due to their relatively high correlation to stocks.
Legal Disclosures

The views expressed in this article are exclusively those of its author(s) as of the date of the article. The views are provided for informational purposes only, are not meant as investment advice, and are subject to change. Investors should consult a financial advisor for advice suited to their individual financial needs. PanAgora cannot guarantee the accuracy or completeness of any statements or data contained in the article. PanAgora disclaims any obligation to provide any updates on the subject in the future.

PanAgora is exempt from the requirement to hold an Australian financial services license under the Corporations Act 2001 in respect of the financial services. PanAgora is regulated by the SEC under U.S. laws, which differ from Australian laws.

---


ii Qian, Edward E., “Diversification returns of leveraged portfolios.” 2012, working paper

iii The result in Equation (3) is an approximation that is only valid when asset returns are relatively small and leverage is moderate. Due to this limitation, results presented in this paper are not intended for portfolios that are levered 20 or 30 to 1. In addition to excessive leverage, managers of these portfolios violate our rebalance assumption - they don’t practice it rigorously. They probably do it to true up leverage in good times when their portfolios increase in value, i.e. buying more risky assets on the way up. As recent experience shows, they certainly do not rebalance when their portfolios lose significant value, since in that case they either go under or get bailed out, by a parent company or taxpayers, rendering rebalance either unattainable or unnecessary.

iv For 2011, when the S&P 500 index returned a 2.1% the leveraged ETFs would have negative returns.