WORKING PAPER JUNE 14, 2006

PanAgora

Aspects of Constrained Long/Short Equity Portfolios

Note:
This article has been accepted for publication in *The Journal of Portfolio Management* in its January 2007 issue.

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Abstract

In this paper, we investigate the benefit and cost of constrained long/short portfolios by relaxing the no-short constraint on actively managed portfolios. Using a simulation under a variety of real-world assumptions, we demonstrate the net benefit of allowing shorting and various optimal amounts of shorting. We show how the maximum information ratio (IR), for a given level of skill, is achieved as a function of the long-only benchmark, the targeted tracking error, and a variety of important cost considerations such as transaction costs and financing costs. Our simulation results indicate that, in general, higher tracking error portfolios require higher leverage ratios to achieve the same level of IR; similarly, higher benchmark concentration requires higher leverage ratios.

Long-Only Handcuffs

Institutional investors today are increasingly searching for sources of consistent alpha in the quest for higher risk-adjusted returns. In the arena of equity investment, this means two things: 1) find more alpha where possible; and 2) undo traditional coupling of alpha (pure skill) with beta (pure risk premium). The well-accepted equity market neutral strategy (pure long/short portfolio) is a clear beneficiary. This strategy delivers more risk-adjusted return, *other things being equal.* But, it lacks the naturally positive, long-term equity risk premium accorded to a positive beta.

A fiduciary can obviously have both — high pure alpha from a manager plus pure beta using derivatives. Many pension fiduciaries, however, have opted for solutions somewhere between the traditional long-only (beta close to 1) strategy and the totally decoupled solution of pure alpha ("ported" to any desired set of beta exposures). This compromise is one reason we see large flows into enhanced index strategies — consistent (albeit lower) alpha, with an embedded beta of 1.

A more recent solution is *some* relaxation of the *long-only constraint* that resides in the traditional investment guideline. In this way the resulting portfolios can invest both long and short, and continue to manage against their respective benchmarks. We refer hereafter to these as "constrained long/short portfolios." For example, the manager might *buy* a 125% exposure in long equity positions and *sell* a 25% exposure in short equity positions, with the net result — 100% long systematic risk. But the total *leverage* to the alpha source is 150% (125% long and 25% short). Although the constrained long/short portfolios might be suboptimal compared to a market neutral portfolio (with derivatives), it offers considerable benefit over *handcuffed* long-only portfolios.

Our purpose in this paper is to remove the handcuffs. In taking them off, our simulations show that we can land more punches and sustain fewer hits. Using a simulation under a variety of real-world assumptions, we demonstrate the net benefits of allowing shorting and various optimal amounts of shorting. We show how the maximum information ratio (IR), for a given level of skill, is achieved as a function of the long-only benchmark, the targeted tracking error, and a variety of important cost considerations.

Off with Handcuffs — Then What?

What are the merits of the constrained long/short portfolios? First, they deliver more alpha potential. Second, they simplify the overall plan's maintenance of beta integrity consistent with policy. Consider the active manager's goal — beat the market-cap weighted benchmark, subject to typical tracking error constraints. The cap-weighted index Goliaths are heavily weighted toward a set of large-cap stocks. For example, the largest 4% of S&P 500 names comprise about 70% of the index weight. In contrast, the smallest 25% comprise only 4% of the index. If the active manager's skill ability is equal across all cap ranges — how can he win? He can't efficiently express his beliefs in specific stocks. With notional limits (no negative weights) on many of the *bad* ones, there is insufficient funding for the *good* ones. For example, managers can only underweight the small stocks by a few basis points when they have a negative forecast. This implies long-only managers can only add real value from their views on small stocks half of the time — when the forecast is positive!

Some ability to short in the constrained long/short portfolios mitigates this drag to varying degrees. Therefore, in theory one should expect managers with this ability to deliver higher risk-adjusted return than their long-only counterparts. But, is the ratio of 125 long to 25 short optimal? On what does the ratio depend? There are key elements underlying the answer. First, what is the benchmark and the risk budget around it? Second, what are the incremental costs? Third, what is the character of the skill underlying the alpha and how strong is it?

Below the Belt

Shorting puts more power in the punch than merely underweighting a long holding relative to a benchmark. There was a time when some investors equated shorting with hitting below the belt. Today, a more relevant issue is the mechanism of shorting, and its inherent cost. Standard financial theory often invokes the concept of a self-financing portfolio. However, leverage is not free. With a $100 endowment, a long-only investor can buy $100 worth of stocks — a leverage ratio of 1:1. With leverage of, say 125/25, for the long side, the investor incurs the following transactions: 1) buying $100 worth of stocks with his own capital, 2) borrowing $25 in cash to buy an additional $25 worth of stocks and, 3) borrowing $25 worth of stocks and short-selling them with $25 cash proceeds. From a practical standpoint, in the third transaction, the lender (prime broker) simultaneously lends $25 to the investor and keeps the $25 proceeds for the short sale as collateral for the short position. Exhibit 1 shows the investor's balance sheet.

Although the broker pays the investor an interest rebate, it is typically lower than the financing cost of borrowing. Therefore, the interest rate spread on the $25 is a net cost that the investor must bear. For example, a spread of 1% carries the additional cost for overall 125/25 portfolios of 0.25% or 25 basis points. Similarly, the additional cost for 150/50 portfolios would be 0.5% or 50 basis points. A summary of the profit and loss statement is shown in Exhibit 2.

The Leverage Ratio of Unconstrained Optimal Portfolios

Although the constrained long/short portfolios remove the long-only constraint, they still constrain the beta and therefore the extent of the overall short positions. Our goal is to analyze the magnitude of the optimal short positions (long/short ratio). An optimal long/short ratio may be 125/25, 150/50, or the like. We begin by deriving the long/short ratio of an unconstrained active long/short portfolio, and then proceed to the constrained portfolios.

We assume the unconstrained active portfolio is market neutral and dollar neutral. From basic portfolio theory, the active weight a_i in a stock is a function of its residual forecast f_i, specific risk σ_i, and a risk aversion parameter for the portfolio λ. Mathematically, we have the optimal active weight determined by $\alpha_i = f_i / \lambda\sigma_i^2$, since residuals are uncorrelated and they are adjusted for market neutral and dollar neutral. The risk aversion parameter is inversely proportional to the targeted tracking error. For high tracking error portfolios, λ is smaller, giving rise to larger optimal active weights. The converse is true of low tracking error portfolios. Suppose the benchmark weight for the stock is b_i. The total portfolio weight in stock i is then

$$w_i = \alpha_i + b_i = \frac{f_i}{\lambda\sigma_i^2} + b_i \qquad (1)$$

Depending on the sign of w_i, the position could be either long ($w_i > 0$) or short ($w_i < 0$). Obviously, if the forecast f_i is positive, the position is long. However, if the forecast f_i is negative, which is equally likely for a market neutral signal, the position is short when

$$f_i < -b_i \lambda \sigma_i^2 \qquad (2)$$

The inequality indicates that the probability of a short position is higher if the product on the right side is small. Thus, we are likely to want a short position for a given stock: 1) the lower the forecast, 2) the smaller the benchmark weight, 3) the smaller the specific risk, 4) the lower the risk aversion parameter, and 5) the higher the target tracking error, all things being equal.

We can calculate the average size of the short position for each stock based on these parameters. (The result is given in the appendix.) We then sum up all the average short positions to get the total short position for the portfolio. In aggregate the influences of these parameters carry over to the portfolio.

In summary, the total short position of an unconstrained optimal portfolio increases with lower average stock-specific risk and higher target tracking error. It turns out it also increases with benchmark concentration. For example, the total short position is lower for an equally weighted benchmark and higher for a market-cap weighted benchmark.

Exhibit 3 shows a set of long/short ratios for portfolios, managed against the S&P 500 Index, as an increasing function of targeted tracking error. In the illustration, we assume that all 500 stocks have 35% specific risk (idiosyncratic volatility). Note that the magnitude of both the long and short positions (thus leverage to the alpha source) increases with the tracking error. The net long minus short stays constant at 100%. When the target tracking error is small, say 0.5%, the total short position is small, and the portfolio is almost identical to a long-only portfolio. When the target tracking error rises to, say 2%, the total short position is about 25% and the total long position is about 125%, with total leverage at 150%. When the target tracking error increases further to 3.5%, the total short position is above 50% and the total long position is above 150%, with total leverage at 220%.

The leverage ratio of optimal portfolios also depends on the distribution of benchmark weights. Following Grinold and Kahn [2000], we model benchmark weights using a scaled lognormal distribution with a concentration index *c* (see Grinold and Kahn [2000], or appendix.) When *c* is zero, the benchmark is equally weighted, with each of the 500 stocks at a 0.20% weight. By comparison, a *c* value of 1.2 (approximately the S&P 500) has the capitalization increasing from well below median to well above median for large-cap stocks.

What is the nature of the optimal long/short portfolio as the benchmark increases in concentration? Exhibit 4 shows the long/short ratios for portfolios, managed against indices with 500 stocks as the concentration rises. We perform the analysis holding the targeted tracking error to 3.5%. The far left point on Exhibit 4 has a *c* value of 0, and corresponds to about 200% leverage. This is an equally weighted benchmark, whereas an S&P 500-like benchmark would require leverage of approximately 220% to achieve a 3.5% tracking error. This illustrates that benchmarks with higher concentration require more leverage. In essence, to maintain a target the tracking error versus a skewed benchmark uses up some of the leverage that could otherwise be devoted to enhancing the pure alpha.

The leverage ratio of unconstrained optimal portfolios also depends on the number of stocks in the benchmark. The Russell 2000 Index, for example, is less concentrated than the S&P 500 Index, with a concentration parameter of 0.8. However, since it has close to 2,000 stocks, its benchmark weights are uniformly smaller than those in the S&P 500 Index. As a result, optimal portfolios managed against the Russell 2000 generally have more short positions, hence higher leverage than optimal portfolios versus the S&P 500 Index. This is true despite the fact that specific risks for Russell 2000 stocks are higher than those in the S&P 500.

The Information Ratio of Long/Short Portfolios — Constant IC

The previous section highlights the fact that unconstrained optimal portfolios have intrinsic long/short leverage ratios, which depend on both active portfolio and benchmark risk characteristics. In theory, these long/short ratios are optimal for given portfolio mandates in terms of maximizing IR. If one chooses, it can be implemented with pure long/short market neutral portfolios plus a derivative for the index exposure. For the constrained long/short portfolios, however, practical constraints such as limiting the individual position magnitudes and a variety of costs serve to reduce the achievable IR.

Some of the constraints are institutional. For example, prime brokers might place limits on the amount of leverage allowed in a portfolio; or for certain stocks borrowing may be difficult. These would reduce the amount of shorting. These types of restrictions are *discrete* in that there are specific quantity limits.

Other restrictions are cost-related, and should be thought of as *continuous*. It just becomes more expensive as we increase the magnitude of shorting, and at some point it creates an excessively high drag on the net IR. Financing cost, mentioned earlier, constitutes one of these. In addition, transaction costs increase as the short positions rise due to increased portfolio turnover. Not only does the number of specific stocks rise, there may also be a need to rebalance more frequently. In fact, shorting requires a greater monitoring effort in the process of trading due to the fat-tailed nature of single stock return distributions and the potential for unrealized losses as short-position prices spike from time to time. In addition, there is the requirement of the management team to *cover* more names, imposing additional research work, particularly for fundamental managers.

What are the optimal solutions for a long/short ratio when we consider all these market imperfections? To understand the net benefits of constrained long/short portfolios, we performed numerical simulations, creating long/short portfolios with differing levels of short positions. Beginning with the limiting case of no shorting, and sequentially relaxing the position limit on any holding, we can find the ratio that maximizes the net IR, all things being equal. (The appendix provides details of the simulation assumptions.) Key factors include the nature of the alpha, the tracking error target, the assumed turnover, the leveraging costs, and the trading costs.

In the simulation analyses, we first calculate the average "paper" excess returns from optimal portfolio weights and returns through time, and then adjust them for these return drags and frictions. In each run, we generate standardized forecasts and actual returns based on the information coefficient (IC). We then calculate excess return of active portfolios, which are managed against a benchmark with a specified concentration index and a series of targeted tracking errors. These simulations are optimized with increasing relaxation of the short constraint applied to each security weight.

Exhibit 5 presents the results for simulations with a target tracking of 3% and an assumed skill IC that has expected value of 0.10 for all stocks over time. The far left column represents long-only, and the far right represents optimally unconstrained. This last portfolio is not totally unconstrained in that we only allow active weights as high as a reasonable 3%. As we move down the body of Exhibit 5, we first present the average magnitude of optimal shorting in the aggregate portfolio (0% to 48%) and average turnover (64% to 94%).

We show the average alpha and standard deviation before considering costs associated with leverage and turnover. This produces a theoretical IR that rises from 1.59 for long-only to 2.24 for the unconstrained simulation. However, there are costs.

We estimate leverage cost and transaction cost, and subtract them from the theoretical excess return. This gives rise to a "net" IR calculated as the ratio of net excess return to the realized tracking error, not the target tracking error. In the case considered here, the two are indistinguishable because we assume the IC is constant. Exhibit 5 shows that portfolio turnover increases with leverage. It averages 64% for the long-only portfolio and about 94% for the unconstrained portfolio. These numbers are based on our assumption of forecast autocorrelation of 0.25[1]. Active short constraints have a dampening effect on portfolio turnover, since they prohibit portfolios from adjusting fully according to changes in forecasts. They also have a negative impact on the investment performance due to suboptimality. It is very interesting to note in Exhibit 5 that the turnover is basically a linear function of leverage.

To calculate the net average alpha, we assume the spread between long financing and short rebate is 1% and a transaction cost of 1% for 100% turnover. These rates are reasonable and conservative estimates. In practice, the financing and rebate spread is subject to negotiation with prime brokers, and the transaction cost depends on many factors such as commission, bid/ask spread, and market impact. Using net average alpha, the long-only portfolio IR drops from 1.59 to 1.38, a decrease of 0.21; the unconstrained optimal portfolio IR drops from 2.24 to 1.77, a much larger decrease (0.47), due to higher leverage cost and transaction cost.

Lastly, we compute both the theoretical and net (after-cost) IR drag, defined as ratio of IR of the constrained to the unconstrained theoretical (and net) portfolios, respectively. For example, the long-only portfolio's theoretical IR drag is 71% of unconstrained IR (1.59 versus 2.24). However, its net IR drag is 78% of unconstrained net IR (1.38 versus 1.77). The constrained portfolio (#6), with an average of 33% short, achieves about 95% of unconstrained net IR (1.68 versus 1.77).

The last row of Exhibit 5 shows the transfer coefficient (Clarke et al. [2002]), defined as the correlation between the active weights in the constrained portfolios and the forecasts. In this case, the transfer coefficients are close to theoretical IR decay but differ from the net IR decay.

Exhibit 6 presents a graphic illustration of the theoretical IR and net IR as we relax the short constraint. We note two features of the graph. First, the rate of increase in IR with relaxing the short constraints is higher in terms of theoretical IR than in terms of net IR. This is due to the higher leverage cost and higher transaction cost associated with less constrained portfolios. Second, both are curves, and not straight lines. The marginal increase in IR seems to be the strongest for long-only portfolios, and it diminishes as the short constraints are relaxed further.

One of the many reasons for the low IR of the long-only portfolios is the inferior allocation of active risk. If a signal has uniform predictive power across stocks of all sizes, then the optimal allocation of active risk should be equal across the size spectrum. But this is not the case for the long-only portfolios since the constraint forces more active risk accorded to stocks with large benchmark weights. Exhibit 7 shows the contribution to active risk from five quintiles of stocks ranked by benchmark weights (rank 1 is the largest stocks and rank 5 is the smallest) for the 11 portfolio simulations. The targeted tracking error is 3%. The long-only portfolio has 45% of risk in the largest quintile and 17% in the second-largest quintile, while the remaining three quintiles each contribute roughly 13%. As we loosen the short constraint, the contribution from the largest quintile decreases while the rest contribute more, until we reach the unconstrained portfolio where all quintiles contribute an equal and optimal amount — 20% to the active risk.

The Information Ratio of Long/Short Portfolios – Stochastic IC

One of the underlying assumptions for the simulations is a constant IC. This assumption, which also underlies some previous results (Grinold and Kahn [2000], Clarke et al. [2002]), however, is often violated in practice. Qian and Hua [2004] show that active investment strategies bring additional risk that is not captured by generic risk models, and as a result, the realized or *ex post* tracking error often exceeds the target or *ex ante* tracking error. We refer to this additional risk as strategy risk, and represent it by the intertemporal variation of IC. The realized tracking error is then a function of standard deviation of IC that consists of both the intertemporal variation and the sampling error. The IR of an active investment strategy is then given by the ratio of average IC to the standard deviation of IC

$$IR = \frac{\overline{IC}}{\text{std}(IC)} \qquad (3)$$

For example, if the intertemporal variation of IC is 0.03, then the standard deviation of IC is

$$\text{std}(IC) = \sqrt{0.03^2 + \frac{1}{N}} = \sqrt{0.03^2 + \frac{1}{500}} = 0.054$$

And the IR of unconstrained portfolios is

$$IR = \frac{0.1}{0.054} = 1.86$$

Exhibit 8 shows the simulation results that take into account the additional intertemporal variation of IC, in this case, at 0.03. First notice that the unconstrained portfolio (#11) has a realized tracking error of 3.62% even though the target is 3%, due to additional strategy risk, and the theoretical IR is 1.86 as we indicated earlier. Second, we note the realized tracking error for the long-only portfolio is 3.25%, closer to the target even though it is still higher. As a result, the long-only theoretical IR is 1.46, smaller than the unconstrained one by 0.42. And as we relax the no-short constraint, the realized tracking error increases. These results indicate that more stringent range constraints have the potential benefit of controlling *ex post* tracking error when there is additional strategy risk. Put differently, relaxing the long-only constraint could potentially lead to higher *ex post* tracking error, and portfolio managers must pay extra attention to risk management.

The other characteristics of the portfolios, such as percent short and turnover, remain roughly the same, so additional costs remain unchanged. However, the net IR is lower in Exhibit 8 than in Exhibit 5 due to higher realized tracking error. Lastly, the differences between IR decay and transfer coefficient grow substantially larger. For instance, the transfer coefficient for the long-only portfolio is 0.70 while the net IR decay is 0.86 and the theoretical IR decay is 0.78. In general, when the strategy risk grows, we find that transfer coefficient can no longer equate with the ratio of constrained IR to the unconstrained IR. It is worth noting that the stochastic IC assumption, which is more realistic than the constant IC assumption, makes a considerable difference in simulation results for the realized tracking error and IRs.

Summary

When relaxing the no-short constraint on actively managed portfolios, one should focus on the benefits and cost of constrained long/short portfolios. Relaxing the no-short constraint broadens investment opportunities that allow portfolio managers to achieve higher risk-adjusted returns. However, there are also additional costs associated with constrained long/short portfolios, including leverage costs and higher transaction costs.

We find that the benefits outweigh the costs under reasonable assumptions. Using a simulation (active portfolio risk of 3%), an unconstrained long/short portfolio has a leverage ratio of 150/50 with a net IR of 1.47, while a constrained long/short portfolio with a leverage ratio of 127/27 has a net IR of 1.41. In general, higher tracking error portfolios require higher leverage ratios to achieve the same level of IR. Similarly, higher benchmark concentration leads to higher leverage ratios. We have performed simulations for other portfolio mandates with different tracking error targets and benchmarks. Our research suggests that the optimal ratio can be quite varied, depending on the mandate and the associated costs of implementation. One important consideration for future research is the *character* of the alpha. What if some managers have higher ICs?

What if their ICs are more predictive for long positions versus short positions, or vice versa? In practice, the relationship between many alpha signals and actual returns is, in fact, nonlinear. In addition, our research (Sorensen, Hua, Qian [2005]) has shown that the information content of many factors and their interactions are contextual, varying across risk dimensions. In all likelihood, the contextual nature of IC affects the results for long/short portfolios.

ASSETS	LIABILITY and EQUITY
Assets	Liability
1a. stocks $100	2b. cash $25
2a. stocks $25	3b. equity $25
3a. cash rebate $25	Equity
	1b. cash $100

Exhibit 1. Leveraged balance sheet.

PROFIT and LOSS

1a. equity return x $100
\+ 2a. equity return x $25
\+ 3a. short rebate interest rate x $25
– 2a. borrowing cost x $25
– 2a. equity return x $25
– 3a. borrowing cost x $25

= Net Return

Exhibit 2. Leveraged profit and loss.

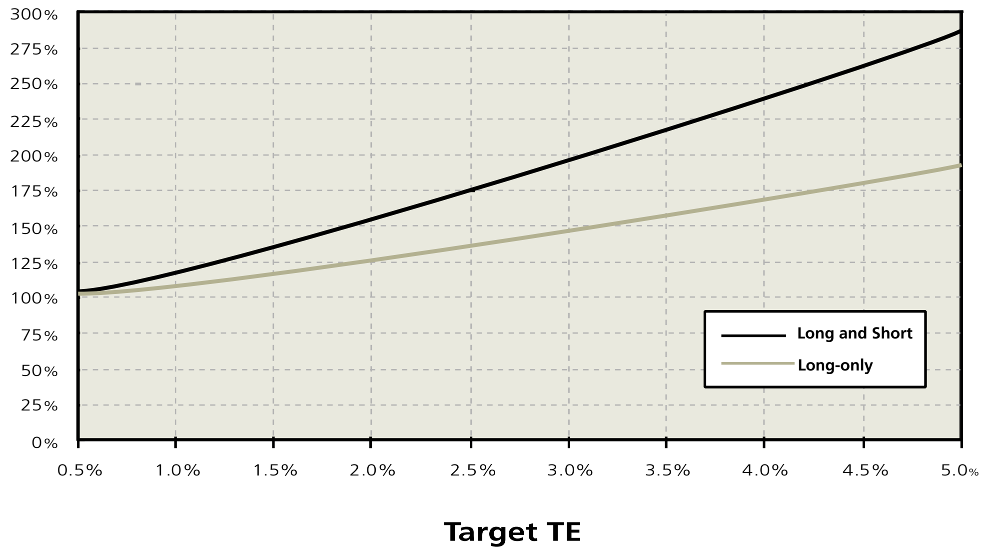

Exhibit 3. The optimal long/short ratios for active portfolios for rising target tracking errors versus S&P 500 Index.

(Number of stocks = 500, and the specific risk = 35% for all stocks.)

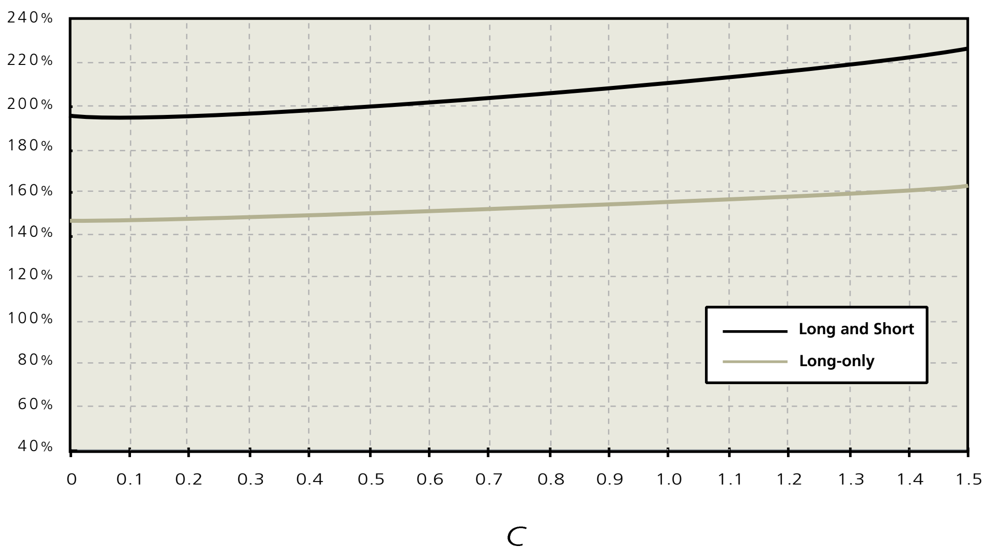

Exhibit 4. Optimal long/short ratios for active portfolios versus increasing benchmark concentrations.

(Number of stocks = 500, and the specific risk = 35% for all stocks.)

	Long-only (Target TE = 3%)										Unconstrained
	1	2	3	4	5	6	7	8	9	10	11
Total Short	0%	9%	15%	21%	27%	33%	38%	43%	46%	47%	48%
Turnover	64%	71%	75%	79%	83%	86%	89%	91%	93%	94%	94%
Leverage Cost	0.00%	0.09%	0.15%	0.21%	0.27%	0.33%	0.38%	0.43%	0.46%	0.47%	0.48%
Transaction Cost	0.64%	0.71%	0.75%	0.79%	0.83%	0.86%	0.89%	0.91%	0.93%	0.94%	0.94%
Avg. Alpha	4.82%	5.30%	5.56%	5.81%	6.05%	6.24%	6.41%	6.54%	6.63%	6.69%	6.72%
Std. Alpha	3.04%	3.03%	3.03%	3.02%	3.01%	3.00%	3.00%	2.99%	2.99%	3.00%	3.00%
Theoretical IR	1.59	1.75	1.84	1.92	2.01	2.08	2.14	2.19	2.22	2.23	2.24
Net Avg. Alpha	4.18%	4.50%	4.66%	4.81%	4.94%	5.05%	5.13%	5.20%	5.25%	5.28%	5.30%
Net IR	1.38	1.48	1.54	1.59	1.64	1.68	1.71	1.74	1.75	1.76	1.77
Theoretical IR Decay	0.71	0.78	0.82	0.86	0.90	0.93	0.95	0.98	0.99	1.00	1.00
Net IR Decay	0.78	0.84	0.87	0.90	0.93	0.95	0.97	0.98	0.99	1.00	1.00
Transfer Coefficient	0.70	0.78	0.82	0.85	0.89	0.92	0.95	0.97	0.98	0.99	1.00

Exhibit 5. Simulation results for long/short portfolios.

(The number of stocks is 500; the benchmark concentration index is 1.2; the target tracking error is 3%; the stock-specific risk is 35%; the average IC is 0.1 with no intertemporal variation; the forecast autocorrelation is 0.25; the leverage cost is 1%; and the transaction cost is 1%.)

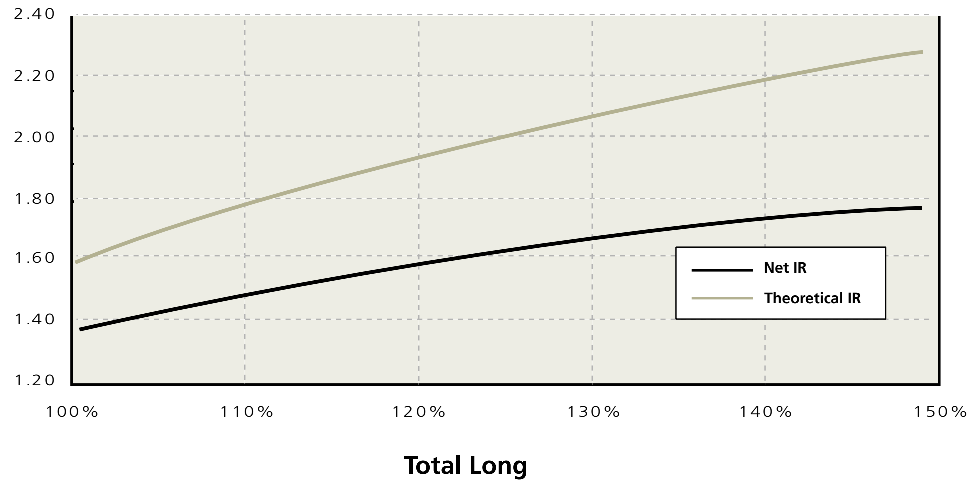

Exhibit 6. The theoretical and net IR shown for Exhibit 5.

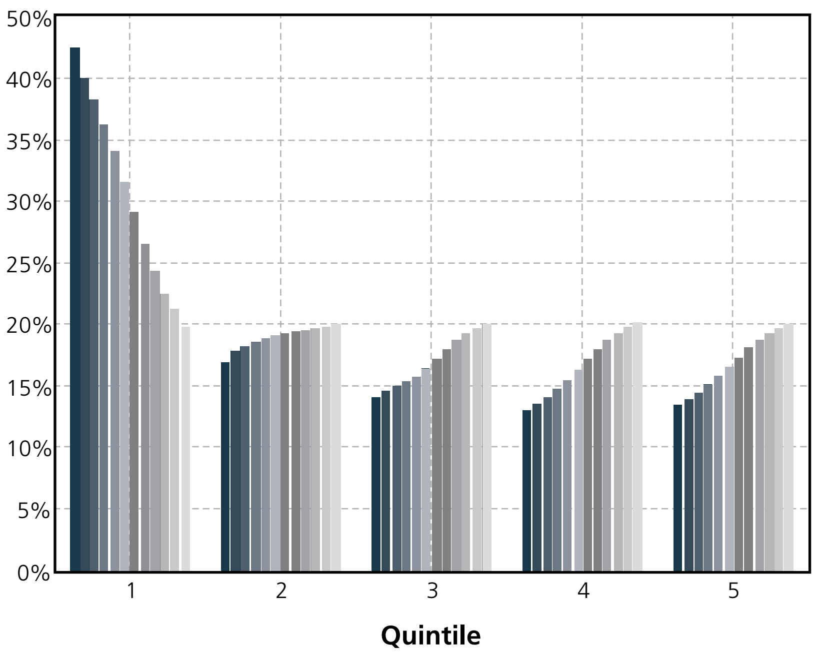

Exhibit 7. Risk contributions from quintiles of stocks according to size.

(The active risk is 3% across 500 stocks, and each quintile has 100 stocks.)

Each vertical bar (color) corresponds to portfolios with a given long/short ratio, from left to right, ranging from the long-only to unconstrained.)

Target TE = 3%

	Long-only										Unconstrained
	1	2	3	4	5	6	7	8	9	10	11
Total Short	0%	9%	15%	21%	27%	33%	38%	43%	46%	47%	48%
Turnover	64%	71%	75%	79%	83%	87%	89%	91%	93%	94%	94%
Leverage Cost	0.00%	0.09%	0.15%	0.21%	0.27%	0.33%	0.38%	0.43%	0.46%	0.47%	0.48%
Transaction Cost	0.64%	0.71%	0.75%	0.79%	0.83%	0.86%	0.89%	0.91%	0.93%	0.94%	0.94%
Avg. Alpha	4.74%	5.23%	5.50%	5.75%	5.99%	6.21%	6.38%	6.53%	6.63%	6.70%	6.75%
Std. Alpha	3.25%	3.33%	3.38%	3.42%	3.47%	3.52%	3.55%	3.58%	3.60%	3.61%	3.62%
Theoretical IR	1.46	1.57	1.63	1.68	1.73	1.76	1.80	1.82	1.84	1.86	1.86
Net Avg. Alpha	4.10%	4.43%	4.60%	4.75%	4.89%	5.01%	5.11%	5.19%	5.25%	5.29%	5.33%
Net IR	1.26	1.33	1.36	1.39	1.41	1.42	1.44	1.45	1.46	1.47	1.47
Theoretical IR Decay	0.78	0.84	0.87	0.90	0.93	0.95	0.96	0.98	0.99	1.00	1.00
Net IR Decay	0.86	0.90	0.92	0.94	0.96	0.97	0.98	0.98	0.99	1.00	1.00
Transfer Coefficient	0.70	0.78	0.82	0.85	0.89	0.92	0.95	0.97	0.98	0.99	1.00

Exhibit 8. Simulation results for long-only portfolios, constrained long/short portfolios, and unconstrained long/short portfolios. The intertemporal variation of IC is 0.03, and all other assumptions are the same as in Exhibit 5.

Appendix

Average Short Positions

We assume the forecast follows a standard normal distribution, i.e., a normalized z score. Then both the active weight and the total weight of stock i follows a normal distribution with a standard deviation

$$s_i = \frac{\sigma_{target}}{\sqrt{N}\,\omega_i} .$$

The mean of total weight is the benchmark weight b_i. The average short-position size can be expressed as a conditional expectation.

$$E\left(w_i + b_i \mid w_i + b_i < 0\right) = \frac{1}{\sqrt{2\pi}\, s_i} \int_{-\infty}^{-b_i} \left(x + b_i\right) \exp\left(-\frac{x^2}{2 s_i^2}\right) dx \tag{1.1}$$

$$= -\frac{s_i}{\sqrt{2\pi}} \exp\left(-\frac{b_i^2}{2 s_i^2}\right) + b_i \cdot \mathrm{cdf}\left(-b_i, 0, s_i\right)$$

The function cdf is the cumulative density function evaluated at $-b_i$ the normal distribution with zero mean and standard deviation s_i.

Simulation of Benchmark Weights

Grinold and Kahn [2000] provided an algorithm to simulate benchmark weights based on scaled lognormal distribution. For a given number of stocks N in the benchmark, a parameter c is used to characterize the concentration of the index. If $c = 0$, the index is equally weighted. As c increases, the index becomes more concentrated. The algorithm has four steps:

1. Discretize the probability interval $(0,1)$ with $p_i = 1 - \frac{i - 0.5}{N},\ i = 1, \cdots, N$
2. Find the value of standard normal variable that has the cumulative probability p_i, i.e., $y_i = \Phi^{-1}(p_i)$, where Φ^{-1} is the inverse of the cumulative density function.
3. Transform y_i to lognormal variable using $s_i = \exp(c y_i)$
4. Scale s_i to obtain benchmark weight $b_i = s_i \Big/ \sum_{i=1}^{N} s_i$

Simulation Assumptions

- **Investment universe and benchmark:** We choose a universe of 500 stocks; portfolios are managed against a 500 stock index, and the index concentration is measured by the parameter c. Stock-specific risk is 35% for all stocks.
- **Tracking error target:** We choose a series of tracking error targets, ranging from 1% to 5%. (Presented here is the case for 3%.)

- **Long/short ratio constraints:** We impose long/short ratio constraint through range constraint on individual stocks. Starting from long-only portfolios, which have constraint on the weights as $w_i \geq 0$, we gradually loosen the constraint to $w_i \geq -s$, where s is the short position allowed in individual stocks. For instance, if $s = 0.1\%$, we can short each stock by a maximum of 10 basis points. As s grows, the total short position grows and the portfolio would approach the unconstrained optimal portfolio. We also set the maximum active weights at +/– 3%. In most cases this 3% limit provides solutions that are optimal for all practical examples.

- **Miscellaneous portfolio constraints:** In addition to targeted tracking error and range constraints on the individual stocks, another portfolio constraint is dollar-neutral constraint — the net active weight is always zero.

- **Alpha forecasts:** We simulated forecast in the form of normally distributed z–scores. We also assume consecutive forecasts have serial autocorrelation $\rho_f = 0.25$, which is one of the factors influencing portfolio turnover. The other factors are target tracking error and leverage ratio.

- **Information coefficient and returns:** The risk-adjusted returns are simulated based on the information coefficient (IC) — the cross sectional correlation coefficient between the forecast and the returns. Two parameters characterize the random nature of IC — the average IC and the standard deviation of IC. The risk-adjusted return is also assumed to be normally distributed, and its cross-sectional dispersion is unity (Qian and Hua [2004]).

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[1] For unconstrained portfolios, the theoretical turnover is a function of stock-specific risks, target tracking error, the number of stocks, and forecast autocorrelation (Qian, Hua, Tilney, 2004).